Objectives for Section 2.3

Recognize Quadratic Equation
Solve Quadratic Equations by:
    Factoring
    Square Root Principle
    Completing the Square
State the Quadratic Formula
State and interpret the Discriminant
Solve Quadratic Equations by:
    Quadratic Formula
Solve Equations that are Quadratic in Form.

2.3 I #s 11, 14, 17, 27, 35

The $b^2-4ac$ part of $\frac{-b \pm \sqrt{b^2-4ac}}{2a}$

2.3 II

Quadratic Function
$f(x) = ax^2 + bx + c$
$f(x) = 3x^2 + 5x - 7$

Quadratic Equation
$ax^2 + bx + c = 0$
$f(x) = 3x^2 + 5x - 7 = 0$
The equation we solve when finding the zeros of f(x).
$f(x) = 0$

The real numbers form a field with multiplication and addition operations.
If A & B are real, then
If $AB = 0$, we have $A = 0$ or $B = 0$.

$(x-2)(x+5) = 0 \Rightarrow$
$x-2=0$ or $x+5=0$
$x=2$ or $x=-5$
$x \in \{-5, 2\}$
Solution Set

This is why we factor.

Zero Product/Factor Principle.

#s 11 -24 Find the zeros of the quadratic function by factoring. What are the *x*-intercepts?

$x^2-3x-28=0$

$(x-7)(x+4)=0$

want factors of $-28$ whose sum (difference) is $-3$

$2 \,|\, 28$
$2 \,|\, 14$
$7$

$28 = 2 \cdot 2 \cdot 7$

$\boxed{(-7)(4)=-28}$

$-7+4=-3$

$x^2-7x+4x-28$

$x(x-7)+4(x-7)$

$(x-7)(x+4)$

$-(15)(24)$

$(3)(5)(2)(2)(2)(3)$

Factors of $-(15)(24)$ whose sum is $2$

$(3)(5)$ $(2)(2)(2)(3)$
$15$ $24$

$(3)(3)(2)$ $(5)(2)(2)$
$-18$ $+$ $20$

Cool!

$= 15x^2+2x-24$

$= \underline{15x^2+20x}-18x-24$ ← Javier

Chris

$= 5x(3x+4)-6(3x+4)$

$=(3x+4)(5x-6)$

$=15x^2+2x-24$ Sweet!

So to solve $15x^2+2x-24=0$ by factoring we have

$(3x+4)(5x-6)=0$

$3x+4=0$ OR $5x-6=0$

$3x=-4$ OR $5x=6$

$x=-4/3$ OR $x=6/5$

$\Rightarrow x \in \{-\frac{4}{3}, \frac{6}{5}\}$

Solve by quadratic formula.

$15x^2+2x-24=0$ $\qquad ax^2+bx+c=0$

$a=15,\ b=2,\ c=-24$ $\qquad x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

$\Rightarrow b^2-4ac = 2^2-4(15)(-24)$

$= 4+1440$

$= 1444$

$$\begin{array}{r} 24 \\ 60 \\ \hline 1440 \end{array}$$

check $\sqrt{1444}=38$, so $38^2=1444$ is a perfect square & $15x^2+2x-24$ factors!

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$=\frac{-2\pm 38}{2(15)}=\frac{\cancel{2}(-1\pm 19)}{\cancel{2}(15)}$$

$$=\frac{-1\pm 19}{15}$$

$\frac{-1+19}{15}=\frac{18}{15}=\frac{6}{5}$

$\frac{-1-19}{15}=\frac{-20}{15}=-\frac{4}{3}$

This says

$15x^2+2x-24=15\left(x-\frac{6}{5}\right)\left(x+\frac{4}{3}\right)$

$=(5)(3)\left(x-\frac{6}{5}\right)\left(x+\frac{4}{3}\right)$

$=5\left(x-\frac{6}{5}\right)(3)\left(x+\frac{4}{3}\right)$

$=(5x-6)(3x+4)$ is factored.

#s 25 - 30 Find the zeros of the quadratic function by the square root method. What are the *x*-intercepts?

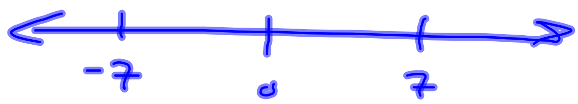
$(-3)^2 = 9 \qquad \sqrt{(-3)^2} = \sqrt{9} = 3 = -(-3)$

$3^2 = 9 \qquad \sqrt{3^2} = \sqrt{9} = 3$

$$\sqrt{x^2} = \left\{\begin{array}{ll} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{array}\right\} = |x|$$

Solve: $|x| = 7$

$x = 7$ OR $x = -7$




$x^2 = 16 \Rightarrow \sqrt{x^2} = \sqrt{16} \Rightarrow |x| = 4 \Rightarrow \boxed{x = \pm 4}$

$x^2 = 16 \Rightarrow x = 4$ No!! ..... yeah, baby.

$x^2 = 7 \Rightarrow x = \pm\sqrt{7}$

$(x+3)^2 = 7 \Rightarrow x+3 = \pm\sqrt{7}$

$\Rightarrow x = -3 \pm \sqrt{7}$

#s 31 - 36 Find the zeros of the quadratic function by the completing the square. What are the *x*-intercepts?

Solving $f(x)=0$

$x^2+6x-7=0$

$\Longrightarrow x^2+6x+3^2=7+9$

Scratch: $\frac{6}{2}=3 \rightsquigarrow 3^2$

$\Longrightarrow (x+3)^2=16$

Seven plus seven. $7+7$ Nathan

$\Longrightarrow x+3=\pm\sqrt{16}=\pm 4$

$\Longrightarrow x=-3\pm 4$: $-3+4=1$, $-3-4=-7$

$\Longrightarrow x\in\{-7,1\}$.

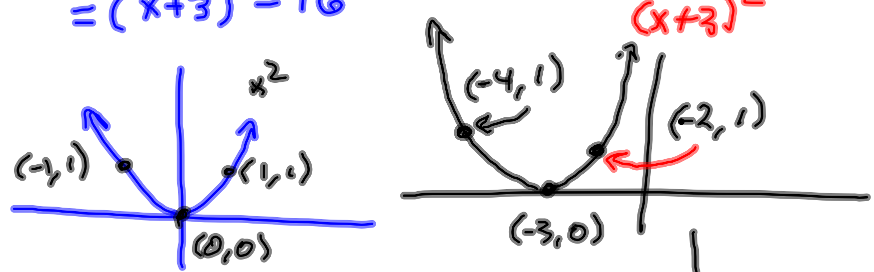
Use completing the square to GRAPH $f(x)=x^2+6x-7$

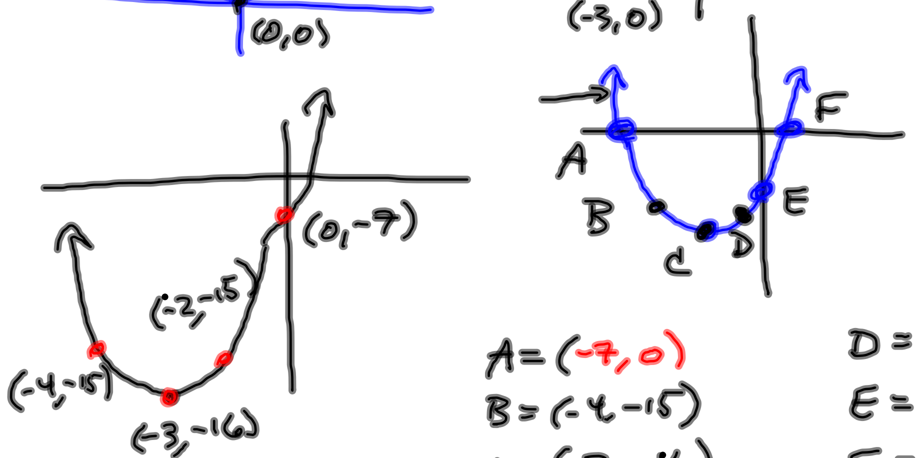
Re-writing $f(x)$ for a graph.

$f(x)=x^2+6x-7$

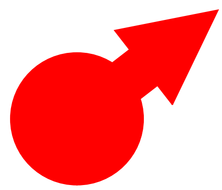
$=x^2+6x+3^2-7-3^2$

$=(x+3)^2-16$




A=(-7,0)
B=(-4,-15)
C=(-3,-16)
D=(-2,-15)
E=(0,-7)
F=(1,0) by previous work.



§2.3 #57

* $x^2 + \sqrt{2}\,x - \frac{1}{2} = 0$ Return to this, maybe.

Equations that are quadratic in form.

$x - 2\sqrt{x} - 15 = 0$

Let $u = \sqrt{x}$ (Then $u^2 = x$ !)

Then $u^2 - 2u - 15 = 0$

$\Rightarrow (u-5)(u+3) = 0$

$\Rightarrow u = 5$ OR $u = -3$ Nevah!

$\Rightarrow \sqrt{x} = 5$ OR $\sqrt{x} = -3$

$(\sqrt{x})^2 = 5^2$

$\boxed{x = 25}$

$(\sqrt{x})^2 = (-3)^2$

$x = 9$

Check:

$\sqrt{9} = +3$

$x^4 - 3x^2 - 18 = 0$

Let $u = x^2$. Then $u^2 = (x^2)^2 = x^{2\cdot 2} = x^4$ ✓

$u^2 - 3u - 18 = 0$

$(u-6)(u+3) = 0$

$u = 6$ OR $u = -3$

$x^2 = 6$ $\quad x^2 = -3$ → Nevah!

$x = \pm\sqrt{6}$

check: $(\sqrt{6})^4 - 3(\sqrt{6})^2 - 18$

$= 36 - 3\cdot 6 - 18$

$= 0$ cool

check: $-\sqrt{6}$ DONE

$f(x) = x^4 - 3x^2 - 18$ is EVEN!

So $f(-x) = f(x)$

So $f(-\sqrt{6}) = f(\sqrt{6}) = 0$

Discriminant

The $b^2-4ac$ part of $\frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$b^2-4ac < 0$ No Real solution
$\sqrt{\text{negative}}$ ain't real.

$b^2-4ac > 0$ 2 distinct, real solutions

$b^2-4ac = 0$ One real zero of MULTIPLICITY 2.
Repeated Root.

$x^2-6x+9=0$

$(x-3)^2=0$

$(x-3)(x-3)=0$

$x-3=0$ OR $x-3=0$

$x=3$ OR $x=3$

$a=1,\ b=-6,\ c=9$

$b^2-4ac=$

$(-6)^2-4(1)(9)$

$=36-36=0$

PERFECT SQUARE TRINOMIAL. It's the square of the binomial $x-3$.